Worked Solutions

Edexcel C3 Paper B

1. (*a*) $\ln(3x+7)=1 \Rightarrow 3x+7=\mathrm{e} \qquad x=\dfrac{\mathrm{e}-7}{3}$ **(3)**

(*b*) $2\mathrm{e}^{2y}+5\mathrm{e}^{y}-3=0$

$\left(2\mathrm{e}^{y}-1\right)\left(\mathrm{e}^{y}+3\right)=0$

$\mathrm{e}^{y}=\dfrac{1}{2} \Rightarrow y=\ln\dfrac{1}{2}\left(\text{or } -\ln 2\right)$ **(5)**

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2. (*a*) any valid pair of $A\&B$ e.g. $A=B=\pi/2$. **(2)**

(*b*) L.H.S. $=\dfrac{2}{\sin 2A}=\dfrac{2}{2\sin A\cos A}=\operatorname{cosec}A\sec A$ **(3)**

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3. (*a*) $\dfrac{2(x+3)+11}{(x-5)(x+3)}=\dfrac{2x+17}{(x-5)(x+3)}$ **(3)**

(*b*) $\dfrac{2x+17}{x^2-2x-15}=1$

$2x+17=x^2-2x-15$

$x^2-4x-32=0$

$(x-8)(x+4)=0$

$x=8 \text{ or } -4$ **(4)**

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4. (*a*) $y=k\ln x \qquad \dfrac{y}{k}=\ln x$

$x=\mathrm{e}^{\frac{y}{k}} \ \therefore \mathrm{f}^{-1}(x)=\mathrm{e}^{\frac{x}{k}}$ **(3)**

(*b*) $\mathrm{gf}(x)=\mathrm{e}^{k\ln x}=\mathrm{e}^{\ln x^k}=x^k$ **(3)**

(*c*) $\mathrm{gf}(2)=16 \ \therefore 2^k=16 \Rightarrow k=4.$ **(2)**

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5. $\sin x=\dfrac{3}{5}$,

$\cos^2 x+\sin^2 x=1$

$\therefore \cos^2 x=1-\dfrac{9}{25}=\dfrac{16}{25}$

$\cos x=-\dfrac{4}{5} \because x \text{ is obtuse.}$

$\therefore \tan x=-\dfrac{3}{4}$

$\cot 2x=\dfrac{1}{\tan 2x}=\dfrac{1-\tan^2 x}{2\tan x}$

$=\dfrac{1-\frac{9}{16}}{2\left(-\frac{3}{4}\right)}$

$=\dfrac{\frac{7}{16}}{\frac{-3}{2}}=\dfrac{-7}{24}$ **(7)**

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6. (*a*) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=x.2\mathrm{e}^{2x}+\mathrm{e}^{2x}.1$

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=0, \quad (2x+1)\mathrm{e}^{2x}=0$

$\Rightarrow x=-\dfrac{1}{2} \quad \text{pt. } \left(-\dfrac{1}{2},-\dfrac{1}{2e}\right)$

$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=2(2x+1)\mathrm{e}^{2x}+\mathrm{e}^{2x}.2$

$x=-\dfrac{1}{2}, \ \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}>0 \ \therefore \text{ min.}$ **(8)**

(*b*) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^0=1$ at origin $\therefore$ tangent is $y=x$. **(2)**

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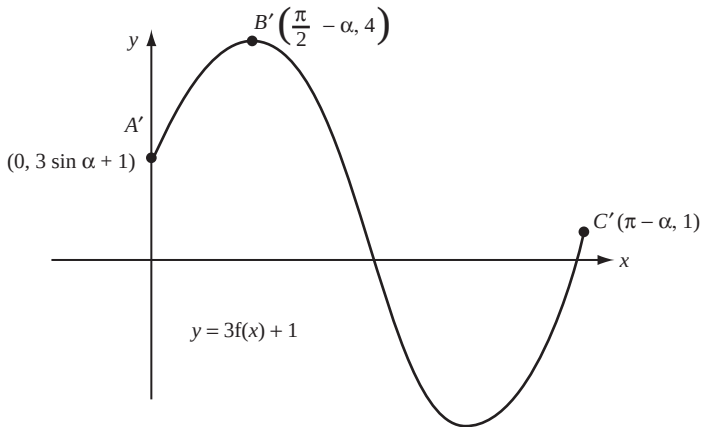
7. (*a*) $A(0, \sin\alpha)$

(*b*) $B\left(\frac{\pi}{2} - \alpha, 1\right)$

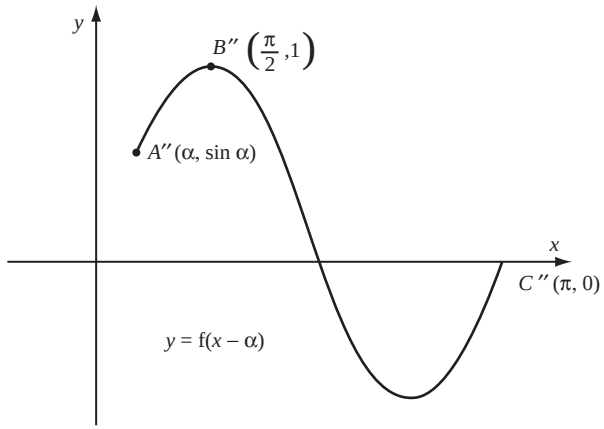
(*c*) $(\pi - \alpha, 0)$ **(3)**

(*b*) (i)



(2)

(ii)



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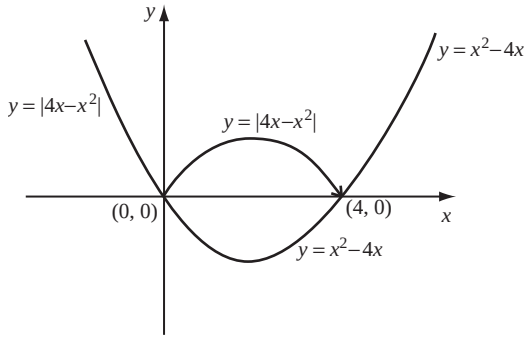
8.



(4)

(*a*) $A = 2\left|\int_0^4 x^2 - 4x \ \mathrm{d}x\right|$

$= 2\left|\left[\frac{x^3}{3} - \frac{4x^2}{2}\right]_0^4\right|$

$= 2\left|\frac{64}{3} - 32\right|$

$= 21\frac{1}{3}$ sq. units **(5)**

(*b*) $y = x^2 - 4x$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - 4$

$x = 4$, gradient $= 4$.

$y = 4x - x^2$,

$\frac{\mathrm{d}y}{\mathrm{d}x} = 4 - 2x$

$x = 4$ gradient $= -4$

$\angle = 2\arctan 4 = 151.9°$ **(8)**
